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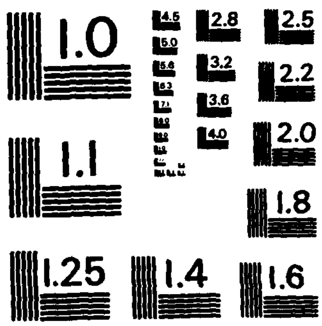

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Parameter Estimation For Static Models
Of The Maypole Hoop/Column Antenna Surface

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P. L. Daniel

E. S. Armstrong

ABSTRACT

We discuss theoretical and numerical results for spline based approximation schemes employed in parameter estimation algorithms for static distributed systems. A specific application involves estimation of parameters in models for the antenna surface in the deployable Maypole Hoop/Column antenna.

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ABSTRACT

We discuss theoretical and numerical results for spline based approximation schemes employed in parameter estimation algorithms for static distributed systems. A specific application involves estimation of parameters in models for the antenna surface in the deployable Maypole Hoop/Column antenna.

INTRODUCTION

Methods for state and parameter estimation in distributed system models for large space structures are of increasing importance in view of the inherent complexity in configuration and material composition of such structures. The deployable Maypole Hoop/Column antenna [12] under development by the Harris Corp. is one such example. The antenna consists of a gold-plated molybdenum reflective mesh surface stretched over a collapsible hoop that supplies the rigidity necessary to maintain the outer circular shape of the antenna. The annular membrane-like surface (100 m. in diameter) surrounds a telescoping column which provides anchoring locations for the flexible mesh and for cables that connect the top end of the mast to the outer hoop and the bottom end of the mast to 48 equally spaced radial graphite cord truss systems woven through the mesh surface. The lower cables, or control "stringers", define the shape of four separate paraboloid dishes located on the surface and thus are of primary importance in the realization of accurate reflector surface control that is needed in communications applications. Fundamental to the implementation of a desired surface configuration (through the adjustment of control stringers) is an analysis of the current antenna configuration from "observations" obtained through sensing devices on the surface; although interesting dynamic state estimation and control problems arise during the initial stages of antenna deployment (from the collapsed "stowed" configuration in the storage compartment of the space shuttle), we will concentrate here on the static identification problems that occur after extended periods of operation. Since it is anticipated that environmental stresses and the effects of aging will contribute to degradation and changes in material properties, the static problem will consist not only of state identification but the estimation of various material parameters and loads related to the structure.

In what follows we describe a static parameter estimation problem associated with a distributed model for the antenna surface. Our efforts have focused on the development of computationally efficient methods for determining stiffness parameters and/or the distributed load on the surface (e.g., through the control stringer/truss system). Although we recognize the possible importance of nonlinear state equations (see, for example [1],[2]) to describe surface displacement and are currently investigating the use of such models, we have chosen a linear distributed model as a starting point for our studies. Our assumed state equation, given here in variational form by

$$(1)\quad \int_0^{2\pi}\int_{R_1}^{R_2} (E\nabla u\cdot\nabla v - fv)r\,dr\,d\theta = 0,$$

relates the vertical displacement u (from hoop level $u = 0$) to the stiffness (elastic) coefficient, $E = E(r,\theta)$, and the applied distributed load, f. In addition to assuming $u(R_2,\theta) = 0$ (along the hoop) we, without loss of generality, take $u = 0$ at the inner circular boundary of the surface ($r = R_1$) since problems with nonhomogeneous boundary conditions may easily be transformed to this form by a change of variable. Finally, $u(r,0) = u(r,2\pi) = g(r)$, where g (or an approximation for g) is assumed known from observations along a single radial cord.

ABSTRACT ESTIMATION PROBLEM

The methods described here are similar in spirit to those for dynamical systems found in [3], [4], [5], [6], [7]; there one reformulates the underlying system (a parameter dependent partial differential equation with boundary and initial conditions) as an abstract evolution equation

$$(2)\quad \begin{aligned} \dot{u}(t) &= A(q)u(t) \\ u(0) &= u_0 \end{aligned}$$

in an appropriately chosen Hilbert space Z. The operator A (usually involving spatial differentiation) may depend on a vector q of parameters to be estimated by, for example, a least squares fit-to-data criterion. For the static antenna problem we take a similar approach in that we reformulate (1) as an abstract variational equation in Z given by

$$(3)\quad \langle A(q)u - f, v\rangle = 0\ ,$$

where the equation holds for all v in some set of test functions.

To construct approximate parameter estimation problems, we choose approximation subspaces Z^N to Z and operators $A^N = P^N A P^N$, where P^N is the orthogonal projection of Z onto Z^N. We then obtain approximating state equations of the form

(4) $\langle A^N(q)u^N - P^N f, v\rangle = 0$

for $u^N \in Z^N$. For finite-dimensional subspaces Z^N, the parameter estimation problem associated with (4) can offer significant computational advantages over the original identification problem, especially when the approximations are such that the dimension of Z^N can be taken small.

Assuming then that the N^{th} approximate parameter *estimation problem* has been solved for some optimal parameter $\bar{q}^N$, we thus have a sequence of parameters $\{\bar{q}^N\}$ for which we have been able to demonstrate convergence (in an appropriate sense) to a solution $\bar{q}$ of the original estimation problem. The convergence arguments, to be described in detail elsewhere, depend on properties of A and Z^N, and require several general assumptions on the parameters and the function f.

SPLINE APPROXIMATIONS FOR THE ANTENNA PROBLEM

To apply the abstract framework described above to the problem of estimating, *for example*, the *stiffness coefficient* $q = E$ in the antenna problem, we first indicate how one computes approximations to the state for a given value of E using standard B-spline spaces ([8], [10], [11]) as the approximation subspaces Z^N. More precisely, in the 1-dimensional problem (where angular symmetry is assumed), let Z^N denote the span of linear spline basis elements $\{B_1^N,\ldots,B_{N-1}^N\}$ defined on a partition of $[R_1,R_2]$ into N equal subintervals, where B_i^N is the standard linear element centered at $R_1 + (R_2 - R_1)i/N$ (we note in this case that each of the basis elements satisfies the homogeneous boundary conditions at R_1 and R_2). Rewriting (4) as

(5) $\langle A(q)u^N - f\ ,\ P^N v\rangle = 0$,

or, equivalently,

(6) $\int_{R_1}^{R_2}(E\nabla u^N \cdot \nabla(P^N v) - f\,P^N v\,)r\,dr = 0$,

we may express u^N in terms of our basis elements, e.g., $u^N = \sum_{i=1}^{N-1} \alpha_i^N B_i^N$, and consider the (matrix) system

(7) $\sum_{i=1}^{N-1} \alpha_i^N \int_{R_1}^{R_2}(E\nabla B_i^N \cdot \nabla B_j^N)r\,dr = \int_{R_1}^{R_2} f\, B_j^N r\,dr,$

$j = 1,\ldots,N-1,$

where $\alpha_i^N = \alpha_i^N(E)$ may be computed for each choice of E by applying standard techniques (e.g., Cholesky decomposition). We note that other spline subspaces (cubic, quintic elements, etc.) may also be used, each of which generates a matrix for the left-hand side of (7) that exhibits a desirable banded structure.

The 2-dimensional antenna problem may be approached in a similar manner, although it poses additional technical difficulties. In this case we choose $Z^N = \mathrm{span}_{i,j}\{B_i^N\Gamma_j^N\}$ where $B_i^N = B_i^N(r)$ and $\Gamma_j^N = \Gamma_j^N(\theta)$ are spline elements (linear, cubic, etc., modified to satisfy appropriate boundary conditions) defined on a partitioning of $[R_1, R_2]$ and $[0,2\pi]$, respectively, into N subintervals. Since the basis elements are now tensor products of spline elements, the 2-dimensional analog of (7) involves the direct product of banded matrices and requires special computational schemes (see [8], [9]).

We turn next to an implementation of the parameter estimation problem for fixed N, noting that the discussion thus far has focused on the solution of (7) (or its *2-dimensional counterpart*) once a choice of the parameter E is given. When E is unknown, conventional optimization schemes (e.g., Levenberg-Marquardt) may be applied to the problem *of* estimating E from observations of the state via, for example, a least squares criterion constrained by the approximating state equation in Z^N. Typically an initial guess E_0^N must be provided; the optimization scheme then generates a sequence $\{E_i^N\}$, requiring that the state equation (7) be solved for each successive iterate E_i^N. Clearly, the efficiency of the estimation effort will depend on the difficulty involved in solving (7) at each step in this process; in the 1-dimensional case, we may simplify this task by picking a representation for E,

$$E(r) = \sum_{k=1}^{K(M)} \gamma_k^M \xi_k^M(r) \ ,$$

where M and ξ_k^M (usually spline elements) are chosen a priori. The advantages to this formulation are twofold: We need only identify a finite number of scalar parameters γ_k^M, $k = 1,\ldots,K(M)$ instead of a <u>function</u> E of r and, since these coefficients will appear in front of the integrals in the left-hand side of (7), the requisite integrations need not be repeated as each parameter iterate is updated. We remark that although the choice of elements ξ_k^M is somewhat arbitrary, it has been our experience that higher order spline elements (e.g., cubic) are more accurate and thus may be effectively utilized with small values of M. A similar representation for E is also desirable in the 2-dimensional problem.

While we shall not emphasize here the theoretical aspects of our investigations, we remark that we have developed a convergence theory for spline based algorithms such as that just described. The fundamental ideas involve variational inequalities and spline approximation estimates for linear and/or cubic elements.

NUMERICAL EXAMPLES

Since we are currently in the process of numerically testing algorithms for the 2-dimensional model, we only present here representative findings for the 1-dimensional problem. In this example, $R_1 = 5$, $R_2 = 50$, and the applied force is taken to be $f(r) = -.0002r^2 + 5$. Since the 1-dimensional counterpart to (1) is equivalent to an ordinary differential equation two-point boundary value problem, an independent method (a multiple "shooting" scheme) was used along with the "true" value of E, $\bar{E}(r) = 2 - \sin\frac{2\pi}{90}(r-5)$, to generate "sample" data, i.e., the values of u at 12 observation points in

$[R_1, R_2]$. From an initial guess of E_0^M, we attempted to find $E^M \approx \bar{E}$ that satisfied a least squares fit-to-data criterion. In addition, we chose a cubic spline representation for E (K(M) = 4) so that our identification problem became that of estimating the coefficients γ_k^M, k = 1,...,4, in that representation. For comparison below, initial guesses and "true" solutions are also expressed in that form.

Example 1: We take N=24 (linear splines) and $E_0^M = 0$.

	γ_1	γ_2	γ_3	γ_4
Initial guess:	0.001	0.001	0.001	0.001
Converged value:	1.424	0.119	0.109	1.514
"True" value:	1.487	0.113	0.113	1.487

Example 2: Here N=24 (linear splines); $E_0^M = 11 - \frac{1}{3}r$.

	γ_1	γ_2	γ_3	γ_4
Initial guess:	3.167	1.667	0.167	-1.333
Converged value:	1.424	0.119	0.109	1.514
"True" value :	1.487	0.113	0.113	1.487

Similar results for a number of other test examples have been obtained.

ACKNOWLEDGEMENTS

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